N°. XXII.

Investigation of the Power of DR. BARKER'S, *Mill, as improved by* JAMES RUMSEY, *with a description of the Mill, by* WM. WARING.

Description *of the* Mill. *Plate* 4. *Fig.* 2.

Read Sept. 21st, 1792.

RT. Is the rotatory; being a tube or trunk into which the water is conveyed by a pipe from the head H, through the neck N and collar C, to the apertures m, n, on contrary sides; where, by its re-action in passing off, it occasions a forcible rotation round the axis or spindle X P, which passes through the lower millstone S and turns the upper one M, or effects other purposes.

Of the proper capacity of the pipe by which the water is conveyed from the head H to the rotatory at N.

Let e= the area of the water's passage at N

h= the perpendicular height of H above N

u= the perpendicular depth of any part of the pipe below H

x= the area at the depth u below H

Then, the areas in the several parts of the pipe (being inversely as the velocities) must be in the inverse subduplicate ratio of the depths below the head; wherefore $\frac{x}{e}=\frac{\sqrt{h}}{\sqrt{u}}$, which gives $x=e\sqrt{\frac{h}{u}}$; so that the pipe must widen towards the head H in the proportion of 1 to $\sqrt{\frac{h}{u}}$; and if the area at any given height be less than $e\sqrt{\frac{h}{u}}$ the water will be obstructed in its passage.

This theorem $(x=e\sqrt{\frac{h}{u}})$ also applies to the pipe of a fire-engine, &c. h being = height of the nozzle from the

 bottom

bottom of the air veſſel toward which the water is uniformly accelerated, u = the height of any other part.

If p = the preſſure of the fluid by its gravity, at the depth h, in a pipe, the area of which is every where = $e\sqrt{\frac{h}{u}}$, it will be, as eh : ue $\sqrt{\frac{h}{u}}$: : p : $p\sqrt{\frac{u}{h}}$, = the preſſure or momentum at the depth u below H.

*Of the initial power of the machine or force with which it begins to move.**

Given, a = area of either aperture; h = height of the water above the centres of the apertures (in feet); w = 62,5 lb. avoirdupois = the Wt. of a cubic foot of water,

Required, I = the initial force, or that with which the machine begins to move.

If we conceive the water preſſing in the tube from O toward I, previous to the opening of the apertures, it is evident it will not produce any motion, becauſe the action againſt each ſide is the ſame: wherefore the preſſure againſt the part m, which is to be removed for an opening, is equal that oppoſed to the ſame area e in the oppoſite direction; now, when the part m is opened, the re-action thereof ceaſes, and the equal impulſe remaining on the contrary ſide e, will be the force required. Viz. a h w for each brachium; conſequently, I = 2hwa = the power with which the rotatory commences its motion. But, as the velocity of rotation increaſes, the relative velocity of the water to that of the tube, and conſequently the power, is diminiſhed, notwithſtanding what is gained by

The

* Benjamin Martin, in his Philoſophia Britannica Vol I. page 217, has attempted to compute the power of ſuch a machine, by the weight and velocity of water emitted per ſecond, &c. without finding the force neceſſary to expel it; but it is not the force accumulated during a ſecond, or any given times, we require, but the power acting continually or as any inſtant conſidered abſtractedly from the idea of time.

The centrifugal force.

Let x=diſtance of any point in the radius from the centre of motion } in feet
r=radius or length of the arm, } in feet
a and w as before,
t=time of a revolution in ſeconds.

Then a will alſo be the area of a ſection of the water paſſing through the tube, at right angles to its direction (or of ſo much of it as we muſt compute the centrifugal force for) which multiplied by the fluxion of x, and by w will be $wa\dot{x}$=the Wt. of the evaneſcent quantity or moving plane $a\dot{x}$, which is the fluxion of the current water in the tube; and, by the doctrine of central forces, as $t^2 : 1.228x :: aw\dot{x} : \frac{1.228awx\dot{x}}{t^2}$= the centrifugal force thereof at x Ft. from the centre of motion, or the fluxion of the whole centrifugal force of the quantity paſſing through either brachium at any time; the fluent of which, when $x=r$, being doubled, is $\frac{76.75ar^2}{t^2}$ =the central force of the water in both arms; which is equal to the augmentation of power thereby occaſioned at the apertures, becauſe fluids preſs equally in all directions. But this force is greatly counteracted by

The Inertia of the Fluid.

The Inertia of the rotatory tube, with the contained fluid, would not continue to reſiſt the moving power, after the velocity became uniform, were the ſame fluid retained therein to which the motion had been at firſt imparted; but as this paſſes off, and there is a continual ſucceſſion of new matter acquiring a motion in the direction of the rotatory, there muſt be a conſtant reaction againſt the inſide of the tube, by the inertia of the fluid, equal to the communicating force. Now this reaction is very

 different

different from that of a fluid confined in the tube when it begins to move, becaufe a particle at the extremity of the tube is not to receive its whole circular motion there, but has gradually acquired it by a uniform acceleration during its paffage along the tube: fo that inftead of the ufual way of computing inertia by the centre of gyration, I muft inveftigate a new theorem for the purpofe (at leaft new to me) which may be thus;

Suppofe a particle P (plate 4 fig. 3,) * moving iniformly in the line and direction C A, while this line has a uniform horizontal motion toward the pofition CB; then P defcribes the common fpiral of Archimedes to Q, &c. and the velocities in P and Q, in the direction of the circumferences paffing through thofe points, are as thofe circumferences, or as their radii CP, CQ, &c. in which ratio are alfo the times of its moving from C to P, Q, &c. And fince the velocities are as the times of moving from C, (as is the cafe of a body falling from reft) the particle P muft be uniformly accelerated, in the direction P n by a conftant equable force, like that of gravity; therefore its reaction againft the moving line CA, by its inertia, muft be the fame in every point from C to A; hence the middle point of the radius is to be confidered as the centre of refiftance in this cafe.

Let $x =$ C P, the diftance in feet of a particle P from the centre at any inftant.

$v =$ the velocity of P per fecond, in the direction of the radius C A.

$c = 3.1416$; a, r, t and w, as before.

Then the moving plane or particle P will be ax, and its weight wax lbs. as before, alfo its velocity $= \dfrac{2cx}{t}$ and the

time

* The velocity muft be uniform if the tube be prifmical; but the effect in this cafe will be the fame if it taper, and the water be accelerated; for the fame quantity in the fame time paffes through (and is acted upon) by every part. Otherwife we fhould ufe the logarithmic fpiral.

time of its acquiring that velocity, *i. e.* of paſſing from C to P, $= \frac{x}{v}$: now the accelerating force neceſſary to communicate a velocity of $\frac{2cx}{t}$ feet per ſecond, to a body weighing awx lb. in $\frac{x}{v}$ ſeconds will be $\frac{cwav\dot{x}}{16t}$ lb. = the fluxion of the inertia, and the fluent, when x becomes = r, will be $\frac{12.272avr}{t}$ lb. = the reſiſtance oppoſed to either brachium, to be eſtimated as if accumulated at $\frac{1}{2}$ r from the centre of motion; conſequently equal to the effect at both apertures when reduced to their diſtance, Q. E. F.

This may be obtained independently of fluxions; by conſidering, that the whole quantity of water (r w a) in the time $\left(\frac{r}{}\right)$ of its paſſing through the rotatory, acquires a velocity $\left(\frac{2crv}{t}\right)$ equal to, and in the direction of, the apertures, as it is carried with the tube out of its natural courſe; to produce which the neceſſary force will be $\frac{12.272avr}{t}$, as before.

Acquired velocity of the water.

The velocity of the water through the apertures at the beginning of rotation is $8\sqrt{h}$ (by the eſtabliſhed principles of hydroſtatics) and, as $2\,wah : \overline{8\sqrt{h}}|^2 = 64h :: 2awh + \frac{76.75ar^2}{t^2} : 64h + \frac{39{,}296r^2}{t^2}$ = the ſquare of the augmented velocity; the ſquare root of which is $8\sqrt{(h + \frac{.614r^2}{t^2})}$ = the acquired velocity of the water.

Proportion

Proportion of the central force to the Inertia.

By fubftituting $8\sqrt{(h+\frac{.614r^2}{ht^2})}$ for v, in $\frac{12.272avr}{t}$ it becomes $\frac{98.176ar^2}{t^2}\times\sqrt{(\frac{ht^2}{r^2}+.614)}$ = the inertia; and, as the central force $\frac{76,75ar^2}{t^2}:\frac{98.176ar^2}{t^2}\times\sqrt{(\frac{ht^2}{r^2}+.614)}::1:1,28\sqrt{\frac{ht^2}{r^2}+.614}=\sqrt{1+(\frac{1.638ht^2}{r^2})}$; that is, the power gained by centrifugal force is to the obftruction occafioned by the inertia, in the proportion of 1 to $\sqrt{(1+\frac{1.638ht^2}{r^2})}$; by which it appears that the latter is the greatter, except when t or h = o, or r infinite; cafes never occuring in practice; and that the longer the brachia, the lefs the fall of water, and the greater the velocity of rotation are, the nearer thefe forces approach the ratio of equality; but as we always find fomething in practical mechanics to prevent our "running into infinitecimals," fo here we are particularly limited; for in the

Adjuftment of the parts and motion.

The centrifugal force fhould not exceed the gravity of the rotating water, or this water would be drawn into the tube fafter than the natural fupply at its entrance, by the velocity proper to that depth; confequently muft lofe the preffure of the column above it: nor fhould the velocity of the apertures, be greater than half that of the water through them; for the apertures being ftill adapted to the velocity, the effluent quantity or number of acting particles is as the time; confequently the momentum is in the fimple

simple ratio of the relative velocity as before demonstrated (at page 146) for the undershot wheel: hence, the greatest effect will be produced when the central force = gravity, and the velocity of the apertures = $\frac{1}{2}$ that of the water; that is, $\frac{76,75ar^2}{t^2} = 2war$; and, $\frac{2cr}{t} = 4\sqrt{h+r}$. from which equations we have the following.

$$\text{Viz.} \left\{ \begin{array}{l} h = 3r = 5t^2 \\ r = 1.63t^2 = \frac{1}{2}h \\ t = v.614r = v\frac{1}{3}h \end{array} \right\}$$ nearly, where we find, h, r, & t, about the constant ratio of 5, 3 and 1.

Yet we may observe here, that while r and t are preserved in a constant ratio, the value of $\frac{76,75ar^2}{t^2}$ and $\frac{12.272avr}{t}$ *i. e.* the central force and inertia must remain the same; so that the brachia may be made to any length at pleasure (not less than $\frac{1}{3}$h) if the time of revolution be proportional, viz. if $t = \sqrt{.614r}$, *i. e.* if the velocity of the apetures be not varied; for a double radius rotating in a double time, or with $\frac{1}{2}$ the angular velocity, has the same absolute velocity at the extremity; and, with the same power, there applied, will produce the same effect. Wherefore, to find,

The moving force and velocity of the Machine, when the effect is a Maximum.

If we put. 614r for t^2 and 3r for h, as before, in the expression $\sqrt{(1+\frac{1.638ht^2}{r^2})}$ it becomes $\sqrt{1+3} = 2$; in which case the resistance of inertia is just double * the central force, or the

* It is demonstrable, that the centrifugal force will be to the inertia, as the velocity of the apertures, is to that of the effluent water; hence also, in the present case, they bear the proportion above stated, exactly.

the gravity of the water in the tube, $= 125ar$, which taken from the impelling force, leaves $62,5(ah+r)-125ar = 62,5a \times \overline{h-r}$ (taking $r = \frac{1}{3}h) = 41\frac{2}{3}ah$ lb. avoirdupois = the real moving force, at the diſtance of the centres of the apertures from the centre of motion. And, by a like ſubſtitution, the velocity $4\sqrt{h+r}$ becomes $4\sqrt{1\frac{1}{3}h} = 4,62\sqrt{h}$ feet per ſecond, Q. E. F.

Area of the apertures.

If $A =$ the area of a ſection of the race, perpendicular to the direction of its motion; $V =$ its velocity per ſecond, both in feet; a and h as before; then it will be, $AV = 8a\sqrt{h}\frac{.614r^2}{t^2}$ cubic feet = the quantity of water emitted per ſecond; hence, $a = \frac{AV}{8.924\sqrt{h}}$ the area proper for one of the apertures.

Scholium.

Were the apertures quieſcent, their area ſhould be enlarged in the proportion of $\sqrt{h}$ to $\sqrt{1\frac{1}{3}h}$, or of 1 to $\sqrt{1\frac{1}{3}}$ to diſcharge the ſame quantity; but then the effluent velocity would be diminiſhed in the ſame ratio; wherefore, $\frac{2wah}{2} = 41\frac{2}{3}ah$, with the ſame velocity, $4,62\sqrt{h}$ as above, will be alſo very nearly the true moving force of a well conſtructed underſhot wheel (J. Smeaton, &c.) Wherefore may be conſidered, in effect, nearly, if not exactly tantamount, when they have the ſame quantity and fall of water; the beſt overſhot being nearly double to either.

From the preceding calculus are deduced the following

Eaſy

Easy practical rules.

1. Make the arm of the rotatory tube, from the centre of motion to the centre of the aperture, of any convenient length, not less than $\frac{1}{3}$ of the perpendicular height of the water's surface above these centres.

2. Multiply the length of the arm, in feet, by .614, and take the square root of the product for the proper time of a revolution in seconds, and adapt the other parts of the machinery to this velocity; or,

3 If at the time of a revolution be given, then, multiply the square of this time by 1.63 for the proportional length of the arm.

4. Multiply together the breadth, depth and velocity per second of the race, and divide the last product by 8.924 times the square root of the height, for the area of either aperture.

5. Multiply the area of either aperture by the height of the head of water, and the product by $41\frac{2}{3}$ (or by 40 on common occasions) for the moving force, estimated at the centres of the apertures in pounds avoirdupois.

6. The power and volicity at the apertures may be easily reduced to any part of the machinery by the common rules of mechanics.